Displacement Current: Fact or Myth?

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Forward: This is about Displacement Current. Maxwell coined the name to describe the term – related to the rate-of-change of the E field -- that permits Ampere's law to apply to both DC and AC circuits. It is often interpreted – in conjunction with another of Maxwell's equations – to show that an E field causes an H field, and vice-versa.

The Displacement Current term is important to antennas. Without it, the mathematical formulas that are derived from Maxwell's Laws do not support radiation. With the term, they do. Nevertheless, after well over a century, we do not have a clear explanation of *how* Displacement Current interacts with an antenna's Electric and Magnetic fields. Put simply, we do not know WHY an antenna radiates! This gap in our understanding is caused – in large part -- by an incorrect interpretation of Displacement Current.

In this article, I shall review a classical "demonstration" of Displacement Current. I shall show that an error in methodology casts doubt on the conclusions. I then shall use basic electronics to show that a capacitor's Displacement Current is nothing more than current flow. After this simple derivation, I shall discuss a more complete analysis that shows that not only does an H field not cause an E field, but that an E field does not cause an H field.

Finally, I shall show that – contrary to previous concepts – Displacement Current has **no impact whatsoever** on the generation of electromagnetic radiation.

The "**Time-Dependent Capacitor:**" The February issue of *antenneX* contains a reprint of a monograph that Dr. Kirk McDonald had posted on his personal website in 2003. The title of his monograph is "Magnetic Field in a Time-Dependent Capacitor." The title is intriguing since capacitors are not "Time Dependent." But the fields contained in capacitors often *are* time dependent, and I suspect that is what he meant to say.

The purpose of his monograph is contained, not in the heading or general text, but in a page-one footnote. There, he states, "The essence of this problem is to show that the magnetic field due to a circular bundle of fine wires, each carrying current I/n is the same as that of a single, fat wire that carries Current I." After solving the problem, McDonald assumes that he may also represent Displacement Current by the same "circular bundle of fine wires." The outcome, of course, is that he "proves" that Displacement Current is capable of generating a magnetic field equivalent to that generated by a "real" conductor.

McDonald constructs a circular capacitor that is fed in the center. The capacitor's dimensions, and the source frequency, are such that the E field is uniform across the plates. This technique is usually called a "quasi-static" analysis. He then asks us to

imagine that the Displacement Current between the plates consists of small filaments, the sum of whose fields satisfies an equation derived from Ampere/Maxwell's law.

Based on these assumptions, he then identifies the magnetic field encircling each hypothetical filament. Next, he observes that the system is symmetrical around the axis of the capacitor. Using this information, he derives a field equation that satisfies the requirement of Ampere/Maxwell. He teaches us that, *everywhere in the space between the plates*, the magnetic field starts at zero at the center and gradually increases with the radius until, at the perimeter it is equal to what it would have been had the gap between the plates been closed.

There is only one small flaw in the conclusion: **After 140 years, no one has ever measured this magnetic field!** One reason why this might be true is that it is extremely difficult to measure small magnetic fields – especially in the presence of stray fields from power leads, testing equipment and other sources.

Another reason might be that the reasoning surrounding hypothesis and conclusion is flawed. A quick review of McDonald's process indicates one such flaw: The observer knows that the system is symmetrical. The filaments do not!

This is important, because it is the filaments that generate the fields, not the observer. And the individual filaments have no way of "knowing" that they are in a structure that is symmetrical. There are two reasons why they do not know it:

1. Filaments are not sentient.
2. The initial conditions of uniform E field and uniform current in the filaments means that the fields associated with every (hypothetical) filament must interact as though the filaments were contained between plates of *infinite size*. Thus, the shape of the plates *cannot* influence the shape of the fields.

To illustrate, let us consider two filaments, A and B separated by a distance that is small with respect to the diameter of the plates, but large with respect to the diameter of the filaments.

Using McDonald's analysis, let us integrate outward from filament A. The magnetic field, according to the paper, will start at zero and circularly expand and strengthen. When it reaches filament B, the magnetic field will – of course -- have a non-zero value. However, if we start from filament B, that is subjected to the same uniform field, the magnetic field at B will *also* start at zero and expand outward until it reaches A, at which point the magnetic field will a non-zero value.

In other words, the two points will simultaneously have zero and non-zero values.

Measured Data: A shrewd reader may ask the following question: If the derivation is flawed, how do we explain the fact that measurements inside a capacitor's gap (and adjacent to the plates) have detected concentric rings of magnetic field that start at (or

near) zero at the center and progressively get stronger as the probe nears the capacitor's rim? Please note, however, that the measured *magnitude* of the fields *does not match* that predicted by Maxwell. Nevertheless, the question is a valid one.

To address this, I would like to use an analogy that I call the "Bumblebee Flaw." Shortly after World War II, scientists decided to simulate the flight of a bumblebee. They used the aerodynamic equivalent of a *quasi-static approximation* based on wing area, wing travel and beats per minute. When they factored in the weight of the bumblebee, they learned that bumblebees couldn't fly! The bumblebee, however, did not know this!

It was only when researchers looked at the *dynamics* of bumblebee flight that they were able to solve the riddle. From this understanding scientists and engineers were able to design better and more efficient airfoils, propellers, variable profile lifting surfaces and a host of other elements.

So, let us attempt to use dynamic analysis on the "Time-Dependant Capacitor." Connect a voltage source and a switch to the capacitor in question. When the switch is actuated, an energy field will begin traveling at light speed along the wire feeding the capacitor. As it does, it will interact with unbound electrons near the surface. Depending on the field's polarity, it causes the near-surface electrons to be either pulled toward the surface or pushed away from it.

The above is consistent with what I have called "Kirk's Teaching," because McDonald was the one that explained this to us over three years ago. When the energy field encounters the back of the capacitor plate, it spreads out to "cover" the outside surface. When the leading edge of the field reaches the edge of the plate, it "turns the corner" and proceeds toward the center.

As the field moves toward the capacitor's center, the leading edge of the field forms an *exact replica* of the shape of the perimeter of the capacitor. At that leading edge, electrons are being pushed into the surface or pulled towards it. This electron motion will be associated with a changing magnetic field having the *same shape* as that of the capacitor's plates.

Although this magnetic field is contained within the plate, we live in an imperfect world. Leakage from that magnetic field is what is measured when a magnetic probe is placed in close proximity to the plate's surface.

A *very* simple derivation of "Displacement Current": The capacitance of a parallel plate capacitor with plates of area A and plate separation of d may be expressed as:

$$C = \varepsilon_R \varepsilon_0 \frac{A}{d} \tag{1}$$

Where ε_R and ε_0 are dielectric and free space permittivity, respectively. The uniform charge, Q stored in a capacitor of capacitance C with Voltage V is as below.

$$Q = CV \tag{2}$$

Divide both sides of equation (2) by d:

$$\frac{Q}{d} = C\frac{V}{d} \qquad (3)$$

But $\frac{V}{d} = E$, where E is the electric field between the plates, so equation (3) becomes:

$$\frac{Q}{d} = CE \qquad (4)$$

Substitute the value for C from equation (1) into equation (4) and we get:

$$\frac{Q}{d} = \varepsilon_R \varepsilon_0 \frac{A}{d} E \qquad (5)$$

Multiply both sides of equation (5) by d, and take the partial derivative with respect to time. The result is:

$$\frac{\partial Q}{\partial t} = \varepsilon_R \varepsilon_0 A \frac{\partial E}{\partial t} \qquad (6)$$

Displacement Current, is defined as $\varepsilon_R \varepsilon_0 E = D$, so equation (6) may be restated as:

$$\frac{\partial Q}{\partial t} = A \frac{\partial D}{\partial t} \qquad (7)$$

Maxwell's well-known adaptation of Ampere's law, to include the effects of Displacement Current is often written as:

$$\nabla x H = J + \frac{\partial D}{\partial t} \qquad (8)$$

Where J is the current density per unit area. Substitute the appropriate values from equation (7) into equation (8) to obtain:

$$\nabla x H = J + A^{-1} \frac{\partial Q}{\partial t} \qquad (9)$$

But $A^{-1} \frac{\partial Q}{\partial t}$ is another way of expressing J. In this case, since the electrical field is

Transverse to the plane of the conductor, we define a new term,

$$J_T = A^{-1} \frac{\partial Q}{\partial t}. \qquad (10)$$

The term "J" has *traditionally* been taken to mean "longitudinal conduction current density," since that is the way that most current flows (through a wire,) To differentiate "regular" J from Transverse J, I suggest that we temporarily rename J as J_L, standing for *longitudinal* current flow. With these re-definitions, we can now rewrite equation (9) in a form that is consistent with the spirit of Ampere:

$$\nabla x H = J_L + J_T \qquad (11)$$

Equations 8, 9 and 11 may be used interchangeably, but only if one understands that the source of the magnetic fields is the motion of charges, $\frac{\partial Q}{\partial t}$. The ***origin and direction*** of the magnetic fields is directly tied to the location and direction of those moving charges.

A more complex analysis of "Displacement Current": In this section, I rely on the work of Dr. Oleg Jefimenko, as published in his books, *Causality, Electromagnetic*

Induction, and Gravitation and *Electromagnetic Retardation and Theory Of Relativity*. Jefimenko notes that Maxwell's Equations represent what is called "action at a distance." That is, there is no provision in any of them to deal with the fact that there is always a finite (albeit sometimes small) time difference between when something happens *here* and the effect of that happening is noted *there*.

Jefimenko starts with these four familiar equations:

$$\nabla \bullet D = \rho \tag{12}$$

$$\nabla \bullet B = 0 \tag{13}$$

$$\nabla x E = -\frac{\partial B}{\partial t} \quad \text{and} \tag{14}$$

$$\nabla x H = J + \frac{\partial D}{\partial t} \tag{15}$$

where E is the electric field vector, D is what he calls the displacement vector, H is the magnetic field vector, B is the magnetic flux density vector, J is the current density vector and ρ is the electric charge density. Jefimenko notes that, since all the above equations connect quantities simultaneous in time, none of them can be causative equations. (A cause must precede an effect.)

But he wonders whether, for example, D rather than $\nabla \bullet D$ might be a consequence of ρ, that E rather than $\nabla x E$ might be a consequence of $\partial B / \partial t$, and H rather than $\nabla x H$ might be a consequence of $J + \partial D / \partial t$?

What he learns is that neither of these ideas has merit. But what else he learns is much more interesting. I will not reproduce his vector analysis, but I refer interested readers to pages 6-13 of Causality. What he teaches us is:

> 1. For equation 14, $\nabla x E$ and $\partial B / \partial t$ have a **common cause.** This is the changing electric current density J.
> 2. For equation 15, $\nabla x H$ and $\partial D / \partial t$ have a **common cause.** This is the changing electric current density J AND the changing ρ.

Here are the resultant E and H field equations:

$$E = \frac{1}{4\pi} \int \left\{ \frac{[\rho]}{r^2} + \frac{1}{rc} \frac{\partial [\rho]}{\partial t} \right\} r_u dv' - \frac{1}{4\pi\varepsilon_0 c^2} \int \frac{1}{r} \left[\frac{\partial J}{\partial t} \right] dv'$$

$$H = \frac{1}{4\pi} \int \left\{ \frac{[J]}{r^2} + \frac{1}{rc} \frac{\partial [J]}{\partial t} \right\} \times r_u dv'$$

While the above are undecipherable without reading Jefimenko's text, one point should be obvious to even the "mathematically challenged:"

1. E is a function entirely of electric charge density and current density.
2. H is a function entirely of current density

Nowhere does Displacement Current appear.

Radiation: Here is what Jefimenko says. "Neither Maxwell's equations nor their solutions indicate an existence of causal links between electric and magnetic fields. Therefore we must conclude that an electromagnetic field is a dual entity always having an electric and magnetic component simultaneously created by their common sources: time variable electric charges and currents."

Summary: For the last 140 years, the rate of change of E fields has been dogmatically accepted as the *source* of the magnetic fields that are hypothesized as existing between the plates of a capacitor, (and everywhere else in an EM environment.) This concept is taught in every EM class worldwide. This is true even though no one has ever been able to measure these mythical between-the-plates fields. The reason for this failure is simple: they are not there.

Using both a simplistic approach (charges moving on and off a capacitor surface) and complex retardation concepts, we come to the same conclusion. Magnetic fields, previously associated with Displacement Current, have a simple – and physically sensible – cause. That cause is charge movement and the rate of change of charges.

Unfortunately, the misinterpretation of Maxwell's equations is not just an academic error of consequence only to scholars. This error has led to at least two failed antenna designs: the CFA and the EH. And those projects have cost millions of wasted dollars, countless wasted hours, and the destruction of many professional reputations.

It is time for instructors everywhere to stop teaching an erroneous concept.

- **30** -

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William C. (Bill) Miller **holds a BSEE from the University of California, Berkeley. He has held Engineering, Product Management and Marketing positions with a variety of well-known companies, including Eitel-McCullough, Ampex, Schlage and Yale.**

In the late 1980's he tired of corporate life. Bill and his Fijian-born wife, Sardha, now own a chain of floral shops in Charlotte, NC. In his worldwide travels, he became familiar with a wide variety of cultures. He speaks

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Bill has been a licensed radio amateur since 1957 and holds an Advanced Class license with the call sign KT4YE. He is an active member of the GARDS, an International Group of compact antenna researchers.