DISPLACEMENT CURRENT DOES NOT EXIST

Part 1: The Capacitor and Displacement Currents

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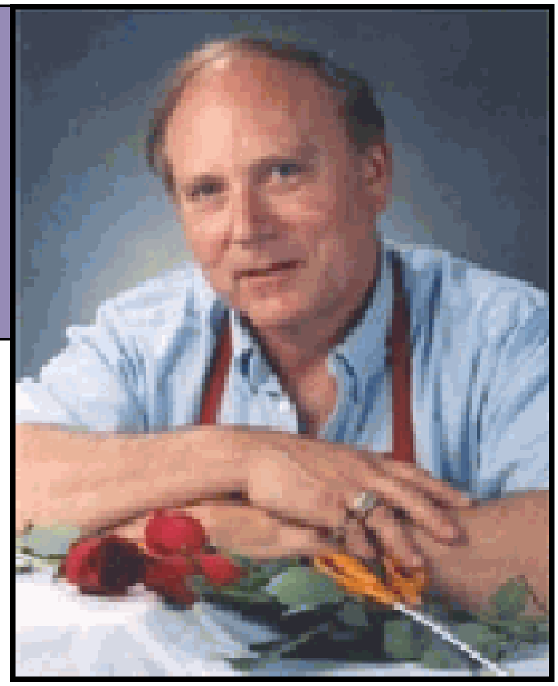


ABSTRACT: Displacement Current is the critical "ingredient" in recent antenna designs such as the CFA and EH. Its existence was postulated in the late 1800's to explain how Alternating Current (AC) could flow between the plates of a capacitor. In this article, we analyze the charge-based model of the capacitor, and attempt to prove that the model is seriously flawed, and perhaps show that Displacement Current does not exist. Its non-existence most likely helps to explain why antennas like the CFA and EH do not perform as predicted by its developers. In this article, we discuss and show that a capacitor is essentially an open-circuit transmission line, and the energy flow is due to the well-understood actions of a Transverse Electromagnetic (TEM) Wave traveling within a transmission line.

What Is A Capacitor? In its simplest embodiment, a capacitor consists of two (usually) closely spaced parallel plates. Each plate is connected to one side of an electrical source – often through a resistor. Starting in the 1800's, a model – based on the concept of charge – was developed for the capacitor. Here are some key concepts:

DC. When a DC voltage is applied to a fully discharged capacitor, the voltage across it will increase in a logarithmic fashion. Using the concept of charge on the plates of a capacitor, the following formula has been derived:

$$V(t)=V\left[1-e^{-\frac{t}{RC}}\right]$$

Where V = the applied voltage, V(t) is the voltage across the capacitor at any time t,
R = the resistor value in ohms, and C = the value of the capacitor in farads.

This formula has been verified experimentally to be extremely accurate.

AC. When an AC voltage is applied to a capacitor, current will flow. There is a 90-degree phase shift between the applied voltage and the current flowing through the capacitor. The formula for capacitive reactance is:

$$Xc = \frac{1}{2\pi f C}$$

Where Xc is the capacitive reactance, f is the frequency in Hertz and C is the capacitance in farads.

As before, this formula has been verified experimentally.

So What's The Problem? Since the equations for the performance of a capacitor have been derived and proven many times over, why should we be concerned? Clearly, we have a model that is theoretically sound, and has been verified experimentally on countless occasions.

The problem can be stated very simply: How in the world can an OPEN CIRCUIT conduct electricity?

Displacement Current to the Rescue! In the late 1800's, James Clark Maxwell – a gifted mathematician — was attempting to develop a unified theory of Electromagnetism. But he was stymied to explain how an AC current could "jump" across an open circuit.

During this period of confusion, he learned of experiments proving that a magnetic field exists at the periphery of a capacitor when AC current is flowing through it. Another pioneer in electricity, Oerstad, had already demonstrated that when a current flows through a wire, it generates a magnetic field.

Maxwell decided that, if ordinary current flowing through a wire causes magnetism, then some type of current *MUST* be flowing through the capacitor. Since the theory of "aether" was prevalent at that time, he speculated that this new type of current was "displacing" the aether in some way. He dubbed this new current, "Displacement Current."

This Displacement Current helped supply the missing component to complete the set of Electromagnetic formulae now known as "Maxwell's Equations."

But Does Displacement Current Exist? Over the years, many proofs have been derived to authenticate and quantify the existence of Displacement Current. For simplicity's sake, most start with a capacitor consisting of a set of circular, parallel plates that are driven at the center.

I am grateful to one of my associates for the following derivation. It is similar to many others that I have encountered in University level classes on Electromagnetic Theory. The problem is not with the derivation. It is with *all* derivations:

H-field in a Capacitor

The capacitor is assumed to be made from two circular plates closely spaced in air. The *z*-axis has its origin at the centre between the plates and lies along the symmetry axis. The feed wires are outside the plates and are connected to the centres of the plates, and lie along the *z*-axis perpendicular to the plates.

The electric field **E** between the plates is an ac field with a frequency whose free-space wavelength is much larger than the plate radius.

In the space between the plates, the displacement current density **D** is given at all points by:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad ..\ 1$$

Maxwell's equation states that:

$$\text{curl } \mathbf{H} = \frac{d\mathbf{D}}{dt} + \mathbf{J} \quad ..\ 2$$

and at all points not in or on the plates and wires **J**=0, so that the following is true in the space between the plates:

$$\text{curl } \mathbf{H} = \frac{d\mathbf{D}}{dt} = \varepsilon \frac{d\mathbf{E}}{dt} \quad ..\ 3$$

With an ac field this simplifies to the following:

$$\text{curl } \mathbf{H} = j\omega\varepsilon \mathbf{E} \quad ..\ 4$$

To evaluate the H-field at a point on the plane *z* = 0, we take a circle *C* of radius *r* centred on the origin. The open surface enclosed by the circle is *S*. For this circle equation 4 is transformed by Stokes's theorem:

$$\oint_C \mathbf{H}.d\mathbf{C} = \iint_S j\omega\varepsilon \mathbf{E}.d\mathbf{S} = j\omega\varepsilon \iint_S \mathbf{E}.d\mathbf{S} \quad ..\ 5$$

Now if we reasonably assume that the electric field is of uniform amplitude at all points in *C*, and parallel to the *z*-axis, and that the magnetic field line at radius *r* follows the perimeter of *C*, the integral (equation 5) simplifies, and **E** and **H** are replaced by their amplitudes *E* and *H*:

$$2\pi r H = \pi r^2 . j\omega\varepsilon E \quad ..\ 6$$

and this gives the solution for the magnetic field at radius *r* inside the capacitor in the *z* = 0 plane:

$$H = \frac{r}{2} j\omega\varepsilon E \quad ..\ 7$$

Application:
If we assume the plates are separated by *d* and are of radius *a*, the voltage between the plates and the capacitance (providing the fringing fields are neglected) are:

$$V = Ed, \quad C = \varepsilon\pi a^2 / d\,, \quad ..\ 8$$

and the current through the capacitor is:

$$I = j\omega CV = j\omega\varepsilon\pi a^2 E \qquad . . 9$$

(Note that the current is independent of the spacing because the E-field is specified, so that larger spacing gives larger voltage, which is balanced out by greater reactance.)

It is possible to estimate the H-field at the edge of the capacitor from the E-field by using the field formula (equation 7) from above:

$$H = \frac{a}{2} j\omega\varepsilon E \,, \qquad . 10$$

and now *E* can be replaced by *I* by using the current formula, (equation 9) so that:

$$H = \frac{a}{2} j\omega\varepsilon \frac{I}{j\omega\varepsilon\pi a^2} = \frac{I}{2\pi a} \,, \qquad . 11$$

This is also the H-field at a distance *a* from the feed wire, at a point well removed from the capacitor. This confirms the field analysis, and it also confirms Maxwell's original postulation of displacement current.

(As a aside, my associate has used an engineering tool that he calls a "field finder" to verify that magnetic fields do, in fact, encircle the center of the structure.

There are Three Problems with the Proof. There are, however, at least three problems with the proof. The problems are not with the mathematics, but with the assumptions. Let us look at the assumptions:

1. The E field is assumed to be uniform at all points within the capacitor. This is not possible because the AC voltage that CAUSES the E field cannot instantaneously disperse from the center to the edge. In fact, since this is a ***charge-based*** model, the speed of dispersal will be limited to the speed of electron flow through metal. That speed is S-L-O-W in comparison with MHz speeds! The E field can *never* be uniform in an AC environment. Even if the AC voltage could disperse to the edges at light speed (believed impossible in a charge-based model) the resultant E field could only be "close" to uniform. In a derivation, "close" may not always provide accurate results.

2. When an AC signal is applied to the center, the voltage at the center will *always* be slightly different from the voltage further out. This will generate secondary E fields that exit perpendicular to each plate and circle around to intersect the same plate at another point. This process further distorts the uniformity of the fields.

3. The proof is based on integrating equation 5 (Stokes Theorem) from the center to some other value, r. It is *assumed* that the magnetic field follows the periphery of the capacitor. However, since the E fields have been assumed to be uniform, there is no

reason for the magnetic fields to follow the periphery – or anything else, for that matter! To illustrate this, if we integrate equation 5 from another starting point to the same end point, r, *we will end up with an entirely different value* for the magnetic field, **H**! Obviously, there is something wrong!

Any one of these flaws would be sufficient to cast doubt on this and all of the other similar derivations. All of the flaws, when taken together, should be enough to negate the derivation. Why hasn't this happened?

It hasn't happened, because there was no real need to do so. Remember that the time constant and reactance equations provide correct answers. The analysis of fields, when the analysis starts at the center, also yields correct answers as long as we don't look too carefully at the assumptions. Besides, the magnetic field *within* a capacitor has always been of little or no importance to engineers.

This lack of importance was true until we started building antennas with capacitors as an integral part of the radiating device, like the CFA and EH. These antennas all attempt to *use* Displacement Current, and they have both suffered from poor performance, a lack of repeatable results, or both.

Are There Any Other Problems? Using the charge-based model of the capacitor, current flow is proportional to the rate of change of the applied voltage. The formula is:

$$I = C\frac{dV}{dt}$$

When you analyze this formula, one fact jumps out. If a "step" of DC voltage is applied to a capacitor, the rate of change of voltage with respect to time becomes infinite. As a result, the initial current also becomes infinite! It is awfully hard to take seriously any physical model that has infinity as one of its end points. (A counter argument is that this can only occur if the source voltage is fed through a source resistor of zero value. But it is much easier to imagine a resistor that is "almost" zero, than it is to imagine a current that is "almost" infinite!)

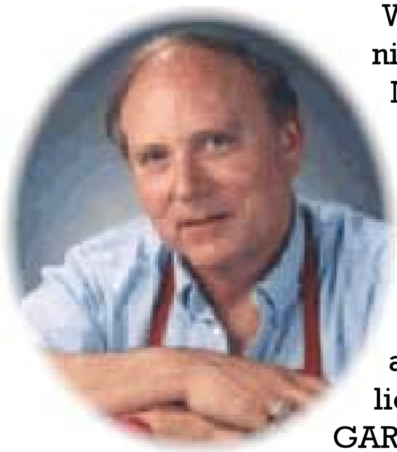
How Important Are These Discrepancies? Many would argue that these errors are of little practical importance. After all, the formulas give the "right" answer, don't they?

In grammar school, many children are taught to use the fraction, 22/7 as a substitute for pi. After all, it gives the "correct" result. But 22/7 has absolutely nothing to do with the relationship between the diameter of a circle and its area. It is simply a number that happens to give the "right" answer.

Can it be that the classical model of the capacitor is as flawed as the idea that 22/7 = pi? Is there a better physical model for the capacitor?

In Part 2, we will explore the transmission line as an alternate model. We shall show that this model eliminates the inconsistencies associated with the "classical" model. We shall learn that this model completely eliminates the need for Displacement Current. **–30-**

Author's Biography



William C. (Bill) Miller holds a BSEE from the University of California, Berkeley. He has held Engineering, Product Management and Marketing positions with a variety of well-known companies, including Eitel-McCullough, Ampex, Schlage and Yale. In the late 80's he tired of corporate life. Bill and his Fijian-born wife, Sardha, now own a chain of floral shops in Charlotte, NC. In his worldwide travels, he became familiar with a wide variety of cultures. He speaks Spanish, French and Portuguese as well as having a working knowledge of German, Italian, Japanese and Hindi. He has been a licensed radio amateur since 1957 and holds an Advanced class license with the call sign KT4YE. He is an active member of the GARDS, an International Group of compact antenna researchers.

antenneX Online Issue No. 67 — November 2002
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